* R M - 6 0 0 1 - R / 1 0 0 *

RM-6001-R

B. E. - I (Sem. - I) (All) Examination

May / June – 2010

Engineering Mathematics - I

Time : **3** Hours] [Total Marks : **100**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **B. E. -1 (SEM. 1 (ALL)**

Name of the Subject :
☛ **ENGINEERING MATHEMATICS - 1**

☛ Subject Code No. : **6 0 0 1** ☛ Section No. (1, 2,.....) : **1&2**

Seat No. :

Student's Signature

(2) All questions are compulsory.

(3) Figures to the right indicate marks.

(4) Draw the figure whenever it is necessary.

SECTION– I

1 (a) Do as directed : **10**

(1) Find Taylor series expansion of $f(x) = \ln x$, at point $a = 1$

(2) If $y = \sin(ax + b)$ then state y_n.

(3) Evalaute $\lim_{x \to y} \frac{x^y - y^x}{x^x - y^y}$

(4) Prove that the logarithm of complex number is a multi valued function.

(5) Using relation between circular and hyperbolic functions prove that $\cosh^2 x - \sinh^2 x = 1$

(b) State and prove Leibnitz's theorem for the n^{th} derivative of the product of two functions. **4**

(c) Attempt the following : **6**

(1) Obtain the first four terms of the Taylor's series of $\cos x$ about $x = \pi/4$.

(2) Find the Maclaurin series expansion of $f(x) = \ln \sec x$ up to six terms.

2 (a) Define radius of curvature for polar form. Show that the radius of curvature at any point (r, θ) on the curve $r^2 = a^2 \sec 2\theta$, is proportional to r^3. **5**

(b) Expand $\log\left(x + \sqrt{x^2+1}\right)$ up to first four terms by Maclaurin's theorem. **4**

(c) Attempt any two of the following : **6**

(1) $\lim_{x \to 2} \left[\frac{1}{x-2} - \frac{1}{\ln(x-1)}\right]$

(2) $\lim_{x \to 0} \frac{2 \tan x \sec x}{x e^x}$

(3) $\lim_{x \to a} (a-x) \tan\left(\frac{\pi x}{2a}\right)$

3 (a) State and prove De Moivre's theorem. **4**

(b) Attempt any two of the following : **8**

(1) If $x_r = \cos\frac{\pi}{2^r} + \sin\frac{\pi}{2^r}$ then prove that $x_1 \cdot x_2 \cdot x_3 \ldots\ldots = -1$.

(2) If α and β are the roots of the equation $x^2 - 2x + 4 = 0$, then prove that $\alpha^6 + \beta^6 = 128$.

(3) Expand $\cos^5\theta \sin^7\theta$ in a series of sine of multiples of θ.

(c) Prove that $\tan^{-1}(\sin\theta) = \cosh^{-1}(\sec\theta)$ **3**

SECTION– II

4 (a) Do as directed : **10**

(1) Define asymptote.

(2) Define linear differential equation of first order and give its general solution.

(3) Sketch the curve $r = a(1+\sin\theta)$

(4) Give the arc length of the curve $x = x(t)$ and $y = y(t)$.

(5) Define order and degree of the ordinary differential equation.

(b) Trace the curve $y^2 = ax^3$ **5**

(c) Attempt any one of the following : 4

(1) Find the area bounded by the curve $x^{2/3} + y^{2/3} = a^{2/3}$

(2) Find the length of the Lemniscate $r^2 = a^2 \cos 2\theta$.

5 (a) Solve any three of the following : **9**

(1) $\left(x^3y^3 + xy\right)dx = dy$

(2) $\left(x^4 - 2xy^2 + y^4\right)dx - \left(2x^2y - 4xy^3 + \sin y\right)dy = 0$

(3) $x(x-y)dy - y(x+y)dx = 0$

(4) $x^2(y+1)dx + y^2(x-1)dy = 0$

(b) Attempt any two of the following : **6**

(1) $(y - px)(p-1) = p$

(2) $y = 2px - p^2$

(3) $xyp^2 - \left(x^2 + y^2\right)p + xy = 0$

6 Attempt any two of the following : **16**

(1) A simple electric circuit contains a resistance of 10 ohms and inductance 4 henries in series with an induced emf of 100 sin 200t volts. If initially current is zero, find the current at time t-0.01.

(2) Formulate MC-model for the spread of technological innovation. Obtain the solution and interpret the result.

(3) State and formulate differential equation model for electric network (RC-circuit with variable emf). Obtain its solution, analyse it and write its interpretation.
